Determination of $30' \times 30'$ marine gravity anomalies from multisatellite altimetry

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Abstract. This paper presents the satellite altimeter data pre-processing and recovery of marine gravity anomalies from the vertical deflections using the inverse Vening Meinesz formula. The formulae for corrections of dynamic oceanic topography (DOT) to the vertical deflections and the model values of mean residual vertical deflections are derived. It shows that the standard deviation (STD) is about $\pm 3.7 \sim \pm 4.4$ mGal by comparing the $30' \times 30'$ gridded gravity anomalies over sea area $82^\circ\mathrm{S} \sim 82^\circ\mathrm{N}$, $0^\circ\mathrm{E} \sim 360^\circ\mathrm{E}$ with that measured by marine gravimeter.

Keywords: gravity anomaly, vertical deflection

1 Introduction

Since the Seasat mission of 1978, global marine gravity anomalies with different accuracies and spatial resolutions have been determined from satellite altimetry. In these gravity derivations, three typical procedures (Hwang et al., 1998) are (1) least-squares collocation; (2) Fourier transform; (3)differentiating along-track sea surface heights to get sea surface gradients, calculating the east and north components of the sea surface gradient on a regular grid, and then recovering gravity anomalies using inverse Vening Meinesz formula. In the paper, the third procedure will be applied and the satellite altimeter data pre-processing presented. Furthermore, the corrections of DOT to the vertical deflections are analyzed and a new formula for the mean vertical deflections from a geopotential model derived. The quality of the derived gravity anomalies will be evaluated by comparison with shipborne gravity data over 15 sampled regions.

2 Data pre-processing

Firstly, the altimeter data will be stacked because the altimeter data are enormous and contain some measurement noises and unmodelled systematic errors. The nominal ground tracks are designed for the altimeter satellite so that the mean sea surface heights at the normal points can be interpolated from the altimeter data along the co-linear tracks by the Stacking technique (Sandwell and Smith 1997), which will improve the accuracies and resolutions of the altimeter data. Secondly, the remove-restore procedure is applied to improve the accuracies of the derived gravity anomalies. According to the procedure, EGM96 (Lemoine et al., 1997) geoidal undulations to degree N are subtracted from the sea surface heights at the normal points, and EGM96 gravity anomalies to degree N are added to the residual derived gravity anomalies to obtain the final marine mean gravity anomalies in the end. Thirdly, a Gaussian low-pass filter will be designed to filter the gradients of the residual sea surface heights (Yale 1995, Wang, 2001) as the simple differentiation of the residual sea surface heights yield or enhance some high frequency noises. Finally, the gradients of the residual sea surface heights are used to compute the vertical deflections at the crossovers and the gridded residual gravity anomalies are predicted using the inverse Vening Meinesz formula with a 1-D FFT method (Hwang C 1998). The above-mentioned data precessing can decrease the influence of the DOT, radial orbit error, atmospheric model error, tidal error, oceanic circulation influence, instrumental error. Till now it is a preferable way to derive the marine gravity anomalies from the altimeter data.

3 Corrections of DOT to the vertical deflection

Although the DOT has the character of long-wavelength and its influence on the vertical deflection is no more than $\pm 0.3''$, it can not be neglected for the determination of high accuracy gravity field. And the corrections of DOT ς to the east $\delta\eta$ and north $\delta\xi$ components of vertical deflection can be written as (Zhang et al., 2000)

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$$\left.\begin{aligned}\delta\xi &= \frac{1}{R}\frac{\partial\varsigma}{\partial\varphi}\\ \delta\eta &= \frac{1}{R\cos\varphi}\frac{\partial\varsigma}{\partial\lambda}\end{aligned}\right\} \quad (1)$$

with

$$\left.\begin{aligned}\delta\xi &= \frac{1}{R}\sum_{n=0}^{K}\sum_{m=0}^{n}\left[c_{nm}\cos m\lambda + d_{nm}\sin m\lambda\right]\frac{\partial\overline{P}_n^m(\sin\varphi)}{\partial\varphi}\\ \delta\eta &= \frac{1}{R\cos\varphi}\sum_{n=0}^{K}\sum_{m=0}^{n}\left[d_{nm}\cos m\lambda - c_{nm}\sin m\lambda\right]\overline{P}_{nm}(\sin\varphi)\end{aligned}\right\} \quad (2)$$

Where R is the mean radius of the earth, λ and φ are the geocentric longitude and latitude, respectively. c_{nm} and d_{nm} are the coefficients of the EGM96 DOT model with degree n and order m, K is the highest degree of the model.

4 Computations of the gridded residual vertical deflections

In theory, the marine mean residual gravity anomalies will be predicted from the residual vertical deflections over the whole earth using the inverse Vening Meinesz formula. The gridded residual vertical deflections over sea area can be obtained from the altimeter data and the reference field. But the mean residual vertical deflections over land or island often have to be supplied by a geopotential model, which can be calculated as

$$\overline{\xi}_{(N+1)\sim 360} = -\frac{1}{\sigma_{ij}}\sum_{n=N+1}^{360}\sum_{m=0}^{n}\left[\left(\overline{C}_n^m\right)^* IC_m^j + \overline{S}_n^m IS_m^j\right] \times\left\{\sqrt{\left(1-\frac{\delta_{0m}}{2}\right)(n+m+1)(n-m)}I\overline{P}_{n,m+1}^i - mI\overline{Q}_{nm}^i\right\} \quad (3)$$

$$\overline{\eta}_{(N+1)\sim 360} = \frac{1}{\sigma_{ij}}\sum_{n=N+1}^{360}\sum_{m=1}^{n} m\left[\left(\overline{C}_n^m\right)^* IS_m^j - \overline{S}_n^m IC_m^j\right]I\overline{B}_{nm}^i \quad (4)$$

where the EGM96 geopotential model (Lemoine et al., 1997) to degree N is used as the reference field, $\left(\overline{C}_n^m\right)^*$ and $\overline{S}_n^m$ are the coefficients of the model,

$$\delta_{0m} = \begin{cases}1, & m=0\\ 0, & m\neq 0\end{cases}, \quad (5)$$

σ_{ij} is the grid area in radian and can be calculated as

$$\sigma_{ij} = \Delta\lambda\left(\sin\varphi_{i+1} - \sin\varphi_i\right) \quad (6)$$

$$IC_m^j = \begin{cases}\Delta\lambda, & m=0\\ \dfrac{2}{m}\sin\dfrac{m\Delta\lambda}{2}\cos m\overline{\lambda}_j, & m>0\end{cases} \quad (7)$$

$$IS_m^j = \begin{cases}0, & m=0\\ \dfrac{2}{m}\sin\dfrac{m\Delta\lambda}{2}\sin m\overline{\lambda}_j, & m>0\end{cases} \quad (8)$$

$\overline{\lambda}_j$ is the geocentric longitude of the grid center; $\Delta\lambda$ is the grid interval in the direction of longitude; $I\overline{B}_{nm}^i$, $I\overline{Q}_{nm}^i$ and $I\overline{P}_{nm}^i$ are the following integrals, respectively.

$$I\overline{B}_{nm}^i = \int_{\varphi_i}^{\varphi_{i+1}}\overline{P}_n^m(\sin\varphi)d\varphi \quad (9)$$

$$I\overline{Q}_{nm}^i = \int_{\varphi_i}^{\varphi_{i+1}}\overline{P}_n^m(\sin\varphi)\sin\varphi d\varphi \quad (10)$$

$$I\overline{P}_{nm}^i = \int_{\varphi_i}^{\varphi_{i+1}}\overline{P}_n^m(\sin\varphi)\cos\varphi d\varphi \quad (11)$$

Using the following recursive relationships of the associated Legendre functions (Abramowitz and Stegun 1964), we have

$$P_n^{m+1}(\sin\varphi) - 2mtg\varphi P_n^m(\sin\varphi) + [n(n+1) - m(m-1)]P_n^{m-1}(\sin\varphi) = 0, \quad m>0 \quad (12)$$

Thus

$$tg\,\varphi\overline{P}_n^m(\sin\varphi) = \frac{1}{2m}\left\{\sqrt{(n-m)(n+m+1)}\overline{P}_n^{m+1}(\sin\varphi)\right\} + \sqrt{\delta_m(n+m)(n-m+1)}\overline{P}_n^{m-1}(\sin\varphi), m>0 \quad (13)$$

where

$$\delta_m = \begin{cases}2, & m=1\\ 1, & m>1\end{cases} \quad (14)$$

Multiplying eq.（13）by $cos\,\varphi$ and integrating the equation, we get

$$\overline{IQ}^{i}_{nm} = \frac{1}{2m}\left\{\sqrt{(n-m)(n+m+1)}\overline{IP}^{i}_{n,m+1} + \right.$$
$$\left. + \sqrt{\delta_m (n+m)(n-m+1)}\overline{IP}^{i}_{n,m-1}\right\}, m>0 \quad (15)$$

Referring to the recursive relationships (Abramowitz and Stegun 1964), we have

$$P^{m}_{n+1}(\sin\varphi) - \sin\varphi P^{m}_{n}(\sin\varphi)$$
$$= (n+m)\cos\varphi P^{m-1}_{n}(\sin\varphi), \quad (16)$$

thus

$$\overline{P}^{m}_{n}(\sin\varphi) = \sqrt{\frac{2n+1}{2n-1}\frac{n-m}{n+m}}\sin\varphi\overline{P}^{m}_{n-1}(\sin\varphi) +$$
$$+\sqrt{\delta_m \frac{2n+1}{2n-1}\frac{n+m-1}{n+m}}\cos\varphi\overline{P}^{m-1}_{n-1}(\sin\varphi), m>0$$
$$(17)$$

Integrating eq. (17), we have

$$\overline{IB}^{i}_{nm} = \sqrt{\frac{2n+1}{2n-1}\frac{n-m}{n+m}}\overline{IQ}^{i}_{n-1,m} +$$
$$+\sqrt{\delta_m \frac{2n+1}{2n-1}\frac{n+m-1}{n+m}}\overline{IP}^{i}_{n-1,m-1}, m>0$$
$$(18)$$

Substituting eq. (15) into eq. (18), we have

$$\overline{IB}^{i}_{nm} = \frac{1}{2m}\sqrt{\frac{2n+1}{2n-1}}\left\{\sqrt{(n-m)(n-m-1)}\overline{IP}^{i}_{n-1,m+1} + \right.$$
$$\left. + \sqrt{\delta_m (n+m)(n-m-1)}\overline{IP}^{i}_{n-1,m-1}\right\}, m>0$$
$$(19)$$

From (Belikov 1991) the fully normalized recurrence relation of row-wise type solving integral $\overline{IP}^{i}_{n,m}$ is

$$\overline{IP}^{i}_{nm} = \frac{n-1}{n+1}\frac{a(n,m)}{a(n-1,m)}\overline{IP}^{i}_{n-2,m} -$$
$$- a(n,m)\frac{1-t^2}{n+1}\overline{P}_{n-1,m}(t)\Big|^{t_2}_{t_1}, m \neq n \quad (20)$$

$$\overline{IP}^{i}_{nn} = \frac{nb(n)b(n-1)}{n+1}\overline{IP}^{i}_{n-2,n-2} +$$
$$+ \frac{t}{n+1}\overline{P}_{n,n}(t)\Big|^{t_2}_{t_1} \quad (21)$$

where $t = \sin\varphi$, $t_1 = \sin\varphi_i$, $t_2 = \sin\varphi_{i+1}$,

$$\begin{cases} a(n,m) = \left[\dfrac{(2n+1)(2n-1)}{(n+m)(n-m)}\right]^{1/2} \\ b(n) = \left(\dfrac{2n+1}{2n}\right)^{1/2}, n>1, b(1) = \sqrt{3} \end{cases} \quad (22)$$

and the starting values for these recurrences can be written as

$$\overline{P}_{00}(t) = 1 \ ,$$
$$\overline{IP}^{i}_{00} = t_2 - t_1, \overline{IP}^{i}_{10} = \frac{\sqrt{3}}{2}\left(t_2^2 - t_1^2\right),$$
$$\overline{IP}^{i}_{11} = \frac{\sqrt{3}}{2}\left(t\left(1-t^2\right)^{1/2} - \arccos(t)\right)\Big|^{t_2}_{t_1}$$

5 Recovery of the marine residual gravity anomalies

The inverse Vening Meinesz formula with a 1-D FFT method (Hwang, 1998) is used to compute the gridded residual gravity anomalies $(\Delta\overline{g})_1$.

$$(\Delta\overline{g})_1 = \frac{\Delta\varphi\Delta\lambda}{4\pi}\gamma_0 F_1^{-1}\left\{\sum_{\varphi_q=\varphi_1}^{\varphi_n}\left[F_1(H'\cos\alpha_{qp})F_1(\xi_{\cos})\right.\right.$$
$$\left.\left. + F_1(H'\sin\alpha_{qp})F_1(\eta_{\cos})\right]\right\} \quad (23)$$

where

$$\left.\begin{aligned} \xi_{\cos} &= \xi_q \cos\varphi_q \\ \eta_{\cos} &= \eta_q \cos\varphi_q \end{aligned}\right\} \quad (24)$$

F_1 denotes the 1D FFT and F_1^{-1} is its inverse, γ_0 is the Earth's mean gravity, GM is the Earth's gravitational constant, ψ_{pq} is the spherical distance between the point p and q, α_{qp} is the azimuth from q to p, ξ_q and η_q are the north and east components of the gridded residual deflection of the vertical at point q, $\Delta\varphi$ is the grid interval in the direction of latitude in radian, $H'(\Delta\lambda)$ is the kernel function of the inverse Vening Meinesz formula.

$$H(\Delta\lambda)=-\frac{\cos\frac{\psi_{pq}}{2}}{2\sin^2\frac{\psi_{pq}}{2}}+\frac{\cos\frac{\psi_{pq}}{2}\left(3+2\sin\frac{\psi_{pq}}{2}\right)}{2\sin\frac{\psi_{pq}}{2}\left(1+\sin\frac{\psi_{pq}}{2}\right)} \tag{25}$$

At zero spherical distance the kernel function $H'(\Delta\lambda)$ becomes singular and the azimuth is undefined. Thus we must account for the innermost zone effect. It can be approximated by

$$\delta\Delta g_p=\frac{S_0\gamma_0}{2}\left(\xi_y+\eta_x\right) \tag{26}$$

where S_0 is the radius of the innermost zone，ξ_y and η_x can be numerically obtained by differentiating ξ and η along the y and x directions, respectively. If the planar grid intervals are Δx and Δy, i.e.

$$\Delta x=R\Delta\lambda\cos\varphi_p$$
$$\Delta y=R\Delta\varphi,$$

The radius of the innermost zone may be approximated by

$$S_0=\sqrt{\frac{\Delta x\Delta y}{\pi}}.$$

We have the approximated representations:

$$\xi_y=\frac{1}{2\Delta y}\left(\xi_{i+1}-\xi_{i-1}\right)$$
$$\eta_x=\frac{1}{2\Delta x}\left(\eta_{j+1}-\eta_{j-1}\right)$$

Where ξ_{i-1} and ξ_{i+1}, η_{j-1} and η_{j+1} are the north and east components of residual vertical deflections at the four discrete grids which are close to the computation point P in the y and x directions, respectively, and the point P is in the discrete grid *ij*. Substituting the above-mentioned relations into eq. (26), we get

$$\delta\Delta g_p=\frac{\gamma_0}{4\sqrt{\pi\Delta\varphi\Delta\lambda\cos\varphi_p}}\left[\Delta\lambda\cos\varphi_p\left(\xi_{i+1}-\xi_{i-1}\right)+\Delta\varphi\left(\eta_{j+1}-\eta_{j-1}\right)\right] \tag{27}$$

After the innermost effect is taken into account, the gridded residual gravity anomalies should be calculated as

$$\left(\Delta\bar{g}\right)_2=\left(\Delta\bar{g}\right)_1+\delta\Delta g_p \tag{28}$$

6 Restoring the mean gravity anomalies

The marine gravity anomalies $\Delta g_{2\sim N}$ at the grid ij from the reference potential model to degree N (Heiskanen , Moritz 1967) can be calculated as

$$\Delta g_{2\sim N}=\gamma_0\sum_{n=2}^{N}(n-1)\sum_{m=0}^{n}\left[\left(\bar{C}_n^m\right)^*\cos m\lambda+\bar{S}_n^m\sin m\lambda\right]\bar{P}_n^m(\sin\varphi') \tag{29}$$

Therefore, the mean gravity anomalies $\Delta\bar{g}_{2\sim N}$ at the grid ij from the reference potential model to degree N can be calculated as

$$\Delta\bar{g}_{2\sim N}=\frac{\gamma_0}{\sigma_{ij}}\sum_{n=2}^{N}(n-1)\left\{\left(\bar{C}_n^0\right)^*\Delta\lambda I\bar{P}_{n0}^i+\right.$$
$$\left.+\sum_{m=1}^{n}\frac{2}{m}\sin\frac{m\Delta\lambda}{2}\left[\left(\bar{C}_n^m\right)^*\cos m\bar{\lambda}_j+\bar{S}_n^m\sin m\bar{\lambda}_j\right]I\bar{P}_{nm}^i\right\} \tag{30}$$

According to the remove-restore procedure, the final marine gravity anomaly can be given by

$$\Delta\bar{g}=\left(\Delta\bar{g}\right)_2+\Delta\bar{g}_{2\sim N} \tag{31}$$

7 Accuracy

We have derived the marine $30'\times30'$ gridded gravity anomalies over the area $82^\circ\mathrm{S}\sim82^\circ\mathrm{N}$ and $0^\circ\mathrm{E}\sim360^\circ\mathrm{E}$ from TOPEX/Poseidon, ERS-1 and ERS-2 altimeter data. The contour map of the marine gravity anomalies from the altimeter data is shown in Figure 1.

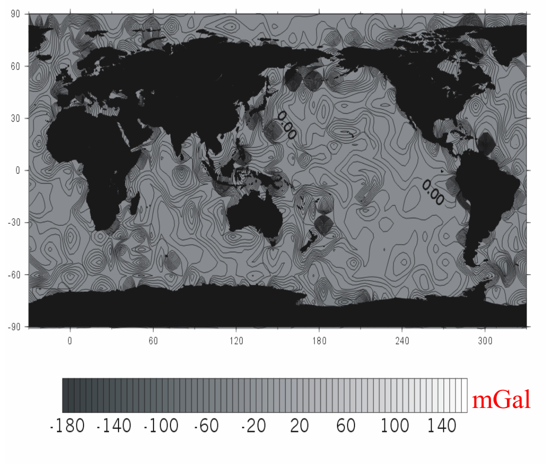

Fig. 1 Gravity anomalies over sea area

Table 1. Information about the satellite altimeter data used in the present analysis

Satellite	ERS-1/2	T/P
Mission	35-day/GM	ERM
Inclination angle(degree)	98.5	66
Repeat period(day)	35/168	10
Number of cycles	50/2	222
Data duration (year)	4.9/0.92	6.1

In order to evaluate the accuracy of the derived gravity anomalies, we compare those with the $30' \times 30'$ ship-measured mean gravity anomalies over 15 sampled sea areas and the statistics of the result are shown in table 2. From table 2 the STD of difference them is $\pm 2.8 \sim \pm 6.0$ mGal and the mean value of the STD is ± 4.8 mGal. Considering the $\pm 2 \sim \pm 3$ mGal accuracy of the ship-measured gravity anomalies, we can conclude that the STD of the derived gravity anomalies is about $\pm 3.7 \sim \pm 4.4$ mGal. It proves that the above-mentioned approaches are significant.

Table 2. Statistics of the difference between satellite-derived and shipborne mean gravity anomalies

Area	Max	Min	Mean	RMS	STD
1	14.7	-13.4	-2.0	±5.2	±4.8
2	7.0	-19.9	-2.2	±6.1	±5.6
3	8.2	-13.3	-2.5	±5.2	±4.5
4	5.6	-8.5	-1.8	±4.3	±3.9
5	5.4	-8.9	-2.1	±4.4	±3.9
6	4.0	-6.9	-3.6	±5.0	±3.4
7	5.7	-4.2	-0.9	±2.9	±2.8
8	6.6	-8.8	-1.2	±4.2	±4.1
9	8.7	-10.6	-0.9	±4.7	±4.6
10	11.7	-11.4	-0.9	±5.2	±5.2
12	8.2	-14.2	-3.0	±5.9	±5.0
13	11.8	-11.3	-2.6	±6.5	±6.0
14	14.6	-14.0	-0.2	±5.3	±5.3
15	6.4	-7.3	-1.5	±3.8	±3.5
Total				±5.1	±4.8

8 Conclusion

This paper describes the procedure and accuracy for global derivation of marine gravity anomalies from multisatellite altimetry. The formulae for corrections of DOT to vertical deflections and the model values of gridded residual vertical deflections are derived, which have improved the accuracy of the derived gravity anomalies. Determination of the marine gravity anomalies from the altimeter data is an important issue in the physical geodesy. From our research we can come to a conclusion that

(1) the influence of the DOT on the vertical deflection should be taken into account in order to improve the accuracy of the determination of gravity anomalies;

(2) the formulae for the model values of the mean gravity anomaly, residual vertical deflection over a gird in the remove-restore procedure are significant to improve the accuracy of the satellite-derived gravity anomalies;

(3) it can decrease the radial orbit error by using the sea surface gradient along the ascending and descending arcs to compute the vertical deflections at the crossovers;

(4) The gridded residual vertical deflections over land or unobserved area can be supplied by the corresponding model value when the inverse Vening Meinesz formula is used to derive the marine gravity anomalies.

In the future further research and test are needed if additional data are available.

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